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UCHINCHI TARTIBLI BITTA SINGULYAR KOEFFITSIYENTGA EGA BO‘LGAN DIFFERENSIAL TENGLAMA UCHUN A_2 CHEGARAVIY MASALA

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Annotatsiya
Ushbu maqolada uchinchi tartibli bitta singulyar koeffitsiyentga ega bo‘lgan xususiy hosilali differensial tenglama uchun to‘g‘ri to‘rtburchak sihada bir chegaraviy masala yechimining yagonaligi va mavjudligi haqidagi teoremalar isbotlangan.

Kalit so‘zlar: *Uchinchi tartibli differensial tenglama, singulyar koeffitsiyent, karrali xarakteristikalar.*

NATIJALAR VA ULARNING TAHLILI

I. Masalaning qo‘yilishi.

Mazkur ishda $D=\{(x,y):0<x<p,\ 0<y<q\}$ sohada ushbu

$$u_{xxx}-u_{yy}-\frac{2\beta}{y}u_y+\mu u=0 \tag{1}$$

tenglamani qaraylik, bu yerda $p>0$, $q>0$, $0<\beta<1/2$, μ – doimiy haqiqiy son.

Ushbu (1) tenglama uchinchi tartibli, xususiy hosilali, singulyar koeffitsiyentli, chiziqli, karrali xarakteristikaga ega tenglamadir.

(1) tenglama uchun quyidagi masalani qo‘yamiz va tadqiq qilamiz:

A_2-masala. D sohada (1) tenglamaning

$$u(x,y)\in C^{3,2}_{x,y}(D)\cap C(\overline{D}),\quad u_{xx},y^{2\beta}u_y\in C(\overline{D})$$

sinfga tegishli bo‘lgan hamda quyidagi

$$u(x,0)=u(x,q)=0,\quad 0\le x\le p \tag{2}$$

$$u_{xx}(0,y)=\psi_1(y),\quad u(p,y)=\psi_2(y),\quad u_x(p,y)=\psi_3(y),\quad 0\le y\le q \tag{3}$$

chegaraviy shartlarni qanoatlantiruvchi yechimini topilsin, bu yerda $\psi_i(y)$, $(i=1,2,3)$ – berilgan funksiyalar.

Ushbu masalada $\beta=0$ bo‘lganda professror Y.P.Apakov va uning shogirti R.A.Umarov tomonidan bir qancha chegaraviy masalalar tadqiq qilingan [1], [2]. Bu masala asosan simmetrik va maxsus xususiyatga ega bo‘lgan (singulyar) o‘tkazuvchan muhitlarda to‘lqinlarning tarqalishi va muvozanat holatini o‘rganish uchun xizmat qiladi.

II. Masala yechimining yagonaligi

Teorema. Agar A_2 masalaning yechimi mavjud bo‘lib, $\mu \ge 0$ bo‘lsa, u holda u yagonadir.

Isbot. Faraz qilaylik, A_2 masala ikkita $u_1(x,y)$ va $u_2(x,y)$ yechimlarga ega bo‘lsin. U holda $u(x,y)=u_1(x,y)-u_2(x,y)$ funksiya bir jinsli A_2-masalaning yechimi bo‘ladi. $\overline{D}$ sohada $u(x,y)\equiv 0$ ekanligini isbotlaymiz.

(1) tenglamaning har ikkala tomonini $y^{2\beta}u(x,y)$ ga ko‘paytiramiz. Natijada quyidagi tenglikka ega bo‘lamiz:

$$y^{2\beta}uu_{xxx}-y^{2\beta}uu_{yy}-2\beta y^{2\beta-1}uu_y+y^{2\beta}u^2=0.$$

Oxirgi ifodani quyidagi ko‘rinishda ham yozish mumkin:

$$\frac{\partial}{\partial x}y^{2\beta}\left(uu_{xx}-\frac{1}{2}u_x{}^2\right)-\frac{\partial}{\partial y}\left(y^{2\beta}uu_y\right)+y^{2\beta}u_y{}^2+y^{2\beta}\mu u^2=0. \qquad (4)$$

(4) ifodani D soha bo‘yicha integrallaymiz:

$$\int_0^q y^{2\beta}\left(u(p,y)u_{xx}(p,y)-\frac{1}{2}u_x^2(p,y)-u(0,y)u_{xx}(0,y)+\frac{1}{2}u_x^2(0,y)\right)dy-$$

$$-\int_0^p y^{2\beta}\left(u(x,q)u_y(x,q)-u(x,0)u_y(x,0)\right)dx+\iint_D y^{2\beta}u_y{}^2dxdy+\iint_D y^{2\beta}\mu u^2dxdy=0.$$

Bir jinsli chegaraviy shartlarni inobatga oladigan bo‘lsak, quyidagi

$$\frac{1}{2}\int_0^q y^{2\beta}u_x^2(0,y)dy+\iint_D y^{2\beta}u_y^2(x,y)dxdy+\iint_D y^{2\beta}\mu u^2(x,y)dxdy=0$$

tenglikka ega bo‘lamiz. Ushbu tenglikda agar

- $\mu=0$ bo‘lsa, $u_y(x,y)=0$ bo‘ladi. Bundan esa $u(x,y)=f(x)$ hosil bo‘ladi. $y=0$ deb olsak so‘ngra bir jinsli chegaraviy shartdan $u(x,0)=0$ ligini hisobga olgan holda $f(x)=0$ bo‘ladi. Demak, $u(x,y)\equiv 0$, $(x,y)\in\overline{D}$.
- $\mu>0$ bo‘lsa uchinchi haddan $u(x,y)\equiv 0$, $(x,y)\in\overline{D}$ ligini olamiz.

Teorema isbotlandi.

III. Masala yechimining mavjudligi

2-teorema. Agar quyidagi $\psi_i(y)\in C^4[0,q]$, $\psi_i^{(4)}(y)\in L_2[0,q]$, $i=\overline{1,3}$ bo‘lganda

$$\psi_i(q)=\psi_i(0)=0,\quad \left|\psi_i'''(0)\right|<\infty \qquad (5)$$

shartlar bajarilsa A_2 masala yechimga ega.

Isbot. A_2 masala yechimini quyidagi

$$u(x,y)=X(x)\cdot Y(y) \qquad (6)$$

ko‘rinishda qidiramiz.

(6) ni (1) tenglamaga qo‘yamiz va $X(x)$, $Y(y)$ funksiyalarga nisbatan quyidagi $Y''+\frac{2\beta}{y}Y'+\lambda^3 Y=0,\quad X'''+\left(\mu+\lambda^3\right)X=0$ oddiy differensial tenglamalarni hosil qilamiz.

(2) shartni inobatga oladigan bo‘lsak, quyidagi

$$Y''(y)+\frac{2\beta}{y}Y'(y)+\lambda^3 Y(y)=0, \tag{7}$$

$$Y(0)=0,\ Y(q)=0 \tag{8}$$

spektral masalani hosil qilamiz, bu yerda λ^3 – ajratish koeffitsiyenti.

Endi {(7), (8)} masalaning noldan farqli yechimi mavjud bo‘ladigan λ parametrning qiymatlarini topish bilan shug‘ullanamiz. Buning uchun λ parametrning uchta holini ko‘rib chiqamiz.

Faraz qilaylik, $\lambda<0$ bo‘lsin. U holda (7) tenglamada $Y(y)=\left(\frac{t}{\sqrt{-\lambda^3}}\right)^{\frac{1}{2}-\beta}p(t),\quad t=\sqrt{-\lambda^3}\,y$ almashtirish bajaramiz va natijada quyidagi

$$t^2p''(t)+tp'(t)-\left(t^2+\nu^2\right)p(t)=0 \tag{9}$$

tenglamaga ega bo‘lamiz, bu yerda $\nu=1/2-\beta$.

(9) tenglamaning umumiy yechimi $p(t)=c_1I_\nu(t)+c_2K_\nu(t)$ ko‘rinishida ekanligi [7], [8] adabiyotlarda ko‘rsatib o‘tilgan, bu yerda c_1 va c_2 – ixtiyoriy o‘zgarmaslar. $I_\nu(t)$ va $K_\nu(t)$ lar esa mos holda ν – tartibli Infeld va Makdonald funksiyalaridir.

Qilingan almashtirishdan foydalanib, y o‘zgaruvchiga qaytadigan bo‘lsak (7) tenglamaning umumiy yechimi quyidagi ko‘rinishda yoziladi:

$$Y(y)=c_1y^{1/2-\beta}I_{1/2-\beta}\left(\sqrt{-\lambda^3}\,y\right)+c_2y^{1/2-\beta}K_{1/2-\beta}\left(\sqrt{-\lambda^3}\,y\right) \tag{10}$$

(10) yechimni (8) chegaraviy shartlarga bo‘ysindiramiz va c_1 hamda c_2 ga nisbatan quyidagi

$$\begin{cases} c_1 0^{1/2-\beta}I_{1/2-\beta}\left(\sqrt{-\lambda^3}\,0\right)+c_2y^{1/2-\beta}K_{1/2-\beta}\left(\sqrt{-\lambda^3}\,y\right)=0,\\ c_1q^{1/2-\beta}I_{1/2-\beta}\left(\sqrt{-\lambda^3}\,q\right)+c_2y^{1/2-\beta}K_{1/2-\beta}\left(\sqrt{-\lambda^3}\,q\right)=0, \end{cases}$$

tenglamalar sistemasini hosil qilamiz. Bu sistemani yechib $Y(y)\equiv 0$ ekanligiga ishonch hosil qilish mumkin. Demak, $\lambda<0$ da {(7), (8)} masalaning noldan farqli yechimi mavjud emas.

$\lambda=0$ da {(7), (8)} masala quyidagi ko‘rinishga ega:

$$\begin{cases} Y''(y)+\frac{2\beta}{y}Y'(y)=0,\\ Y(0)=0,\ Y(q)=0. \end{cases}$$

Oxirgi tenglamaning umumiy yechimi $Y(y)=c_3y^{1-2\beta}+c_4,\quad c_3,c_4=const,$ ko‘rinishida aniqlanadi. Bu funksiyani (8) shartlarga bo‘ysundiramiz va $\lambda=0$ da {(7), (8)} masalaning noldan farqli yechimi mavjud emasligiga ishonch hosil qilish mumkin.

Faraz qilaylik $\lambda>0$ bo‘lsin. U holda $Y(y)=\left(t/\sqrt{\lambda^3}\right)^{1/2-\beta}p(t),\quad t=\sqrt{\lambda^3}y$ almashtirish bajarish natijasida quyidagiga ega bo‘lamiz:

$$t^2p''(t)+tp'(t)+\left(t^2-\nu^2\right)p(t)=0, \tag{11}$$

bu yerda, $\nu=1/2-\beta$. (11) tenglamaning umumiy yechimidan va kiritilgan almashtirishlardan foydalanib (7) tenglamaning umumiy yechimi

$$Y(y)=c_5y^{1/2-\beta}J_{1/2-\beta}\left(\sqrt{\lambda^3}y\right)+c_6y^{1/2-\beta}J_{\beta-1/2}\left(\sqrt{\lambda^3}y\right),\ c_5,c_6=const$$

ko‘rinishda bo‘ladi, bu yerda $J_\nu(x)-\nu$ indeksli birinchi tur Bessel funksiyasi.

Bu umumiy yechimni (8) shartlarga bo‘ysundiramiz:

$$\begin{cases} c_5y^{1/2-\beta}J_{1/2-\beta}\left(\sqrt{\lambda^3}y\right)\Big|_{y=0}+c_6y^{1/2-\beta}J_{\beta-1/2}\left(\sqrt{\lambda^3}y\right)\Big|_{y=0}=0, \\ c_5q^{1/2-\beta}J_{1/2-\beta}\left(\sqrt{\lambda^3}q\right)+c_6q^{1/2-\beta}J_{\beta-1/2}\left(\sqrt{\lambda^3}q\right)=0. \end{cases}$$

Birinchi tenglikdan $c_6=0$ ekanligi kelib chiqadi. Ikkinchi tenglikdan esa $J_{1/2-\beta}\left(\sqrt{\lambda^3}q\right)=0$ tenglamani hosil qilamiz. Bu tenglamaning musbat ildizlarini $\sqrt{\lambda_n^3}q=\sigma_n$ deb belgilab, λ -parametrning qiymatlarini, ya’ni masalaning xos qiymatlarini $\lambda_n=\left(\sigma_n/q\right)^{2/3},\ n\in N$ ko‘rinishida topamiz.

Bu xos qiymatlarga mos keluvchi {(7), (8)}masalaning xos funksiyalari $Y_n(y)=c_5y^{1/2-\beta}J_{1/2-\beta}\left(\sqrt{\lambda^3}y\right)$ ko‘rinishda aniqlanadi. Bu funksiyalar sistemasini normallashtirib quyidagiga ega bo‘lamiz:

$$Y_n(y)=\frac{\sqrt{2}}{qJ_{3/2-\beta}(\sigma_n)}y^{1/2-\beta}J_{1/2-\beta}\left(\frac{\sigma_ny}{q}\right). \tag{12}$$

[8] adabiyotdagi ma’lumotlarga muvofiq (12) funksiyalar sistemasi $L_2[0,q]$ fazosida $y^{2\beta}$ vazn bilan to‘la ortonormal sistemani tashkil qiladi.

Endi (7) tenglamaning tadqiqoti bilan shug‘ullanamiz. (7) xarakteristik tenglama bitta haqiqiy $\tilde{k}_1=-\sqrt[3]{\lambda_n^{\,3}+\mu}$ va ikkita kompleks $\tilde{k}_{2,3}=\left(\frac{1}{2}\pm\frac{\sqrt{3}}{2}i\right)\sqrt[3]{\lambda_n^{\,3}+\mu}$ ildizlarga ega bo‘lsa, uning umumiy yechimi quyidagi ko‘rinishga ega bo‘ladi:

$$X_n(x)=C_{1n}e^{-k_nx}+e^{\frac{1}{2}k_nx}\left(C_{2n}\cos\frac{\sqrt{3}}{2}k_nx+C_{3n}\sin\frac{\sqrt{3}}{2}k_nx\right), \tag{13}$$

bu yerda $C_{jn}-$ ixtiyoriy o‘zgarmaslar, $k_n=\sqrt[3]{\lambda_n^{\,3}+\mu}$. (6) formulaga muvofiq

$$u_n(x,y)=\left[C_{1n}e^{-k_nx}+e^{\frac{1}{2}k_nx}\left(C_{2n}\cos\frac{\sqrt{3}}{2}k_nx+C_{3n}\sin\frac{\sqrt{3}}{2}k_nx\right)\right]Y_n(y), \qquad (14)$$

funksiyalar (1) tenglamaning xususiy yechimlari bo‘lib, ular (2) chegaraviy shartlarni qanoatlantiradi. (1) Tenglamaning chiziqliligi va bir jinsliligidan xususiy yechimlarining yig‘indisi ham (1) tenglamaning yechimi bo‘ladi. Shuning uchun A_2 masalaning yechimi quyidagi ko‘rinishda ifodalanadi:

$$u(x,y)=\sum_{n=1}^{\infty}\left[C_{1n}e^{-k_nx}+e^{\frac{1}{2}k_nx}\left(C_{2n}\cos\frac{\sqrt{3}}{2}k_nx+C_{3n}\sin\frac{\sqrt{3}}{2}k_nx\right)\right]Y_n(y), \qquad (15)$$

$$u_{xx}(0,y)=\psi_1(y),\quad u(p,y)=\psi_2(y),\quad u_x(p,y)=\psi_3(y),\quad 0\le y\le q \qquad (3)$$

(15) qator orqali aniqlanadigan $u(x,y)$ funksiyadan (3) chegaraviy shartlarning bajarilishini talab qilsak, quyidagini olamiz:

$$\begin{cases} A_{1n}=k_n^2\left(C_{1n}-\dfrac{1}{2}C_{2n}+\dfrac{\sqrt{3}}{2}C_{3n}\right), \\ A_{2n}=C_{1n}e^{-k_np}+e^{\frac{1}{2}k_np}\left(C_{2n}\cos\dfrac{\sqrt{3}}{2}k_np+C_{3n}\sin\dfrac{\sqrt{3}}{2}k_np\right), \\ A_{3n}=-k_nC_{1n}e^{-k_np}+k_ne^{\frac{1}{2}k_np}\left(C_{2n}\cos\left(\dfrac{\sqrt{3}}{2}k_np+\dfrac{\pi}{3}\right)+C_{3n}\sin\left(\dfrac{\sqrt{3}}{2}k_np+\dfrac{\pi}{3}\right)\right), \end{cases} \qquad (16)$$

bu yerda

$$\begin{aligned} u_{xx}(0,y)=\psi_1(y)&=\sum_{n=1}^{\infty}A_{1n}y^{\frac{1}{2}-\beta}J_{\frac{1}{2}-\beta}\left(\frac{\sigma_ny}{q}\right), \\ u(p,y)=\psi_2(y)&=\sum_{n=1}^{\infty}A_{2n}y^{\frac{1}{2}-\beta}J_{\frac{1}{2}-\beta}\left(\frac{\sigma_ny}{q}\right), \\ u_x(p,y)=\psi_3(y)&=\sum_{n=1}^{\infty}A_{3n}y^{\frac{1}{2}-\beta}J_{\frac{1}{2}-\beta}\left(\frac{\sigma_ny}{q}\right). \end{aligned} \qquad (17)$$

(17) dan ko‘rinib turibdiki, A_{in} lar $\psi_i(y)$ funksiyalarning $(0,q)$ oraliqda Furye qatoriga yoyilishidagi Furye koeffitsientlaridir, ya’ni:

$$A_{in}=\frac{\sqrt{2}}{qJ_{\frac{3}{2}-\beta}(\sigma_n)}\int_0^q\psi_i(\eta)\eta^{\frac{1}{2}-\beta}J_{\frac{1}{2}-\beta}\left(\frac{\sigma_n\eta}{q}\right)d\eta \qquad (18)$$

Shunday qilib, C_{in} $(i=1,2,3)$ koeffitsientlarni aniqlash uchun biz (17) tenglamalar sistemasini hosil qildik. Ushbu sistemaning yechimi

$$\Delta = k_n^3 \cdot \begin{vmatrix} 1 & -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ e^{-k_n p} & \alpha e^{\frac{1}{2}k_n p} - e^{-k_n p} & \beta e^{\frac{1}{2}k_n p} \\ -e^{-k_n p} & \gamma e^{\frac{1}{2}k_n p} e^{-k_n p} & \delta e^{\frac{1}{2}k_n p} \end{vmatrix} =$$

$$= \sqrt{3}k_n^3 \left[\frac{1}{2} e^{k_n p} + e^{-\frac{1}{2}k_n p} \cos\left(\frac{\sqrt{3}}{2} k_n p \right) \right],$$

bu yerda

$$\alpha = \cos\frac{\sqrt{3}}{2}k_n p,\ \theta = \sin\frac{\sqrt{3}}{2}k_n p,\ \gamma = \cos\left(\frac{\sqrt{3}}{2}k_n p + \frac{\pi}{3} \right),\ \delta = \sin\left(\frac{\sqrt{3}}{2}k_n p + \frac{\pi}{3} \right).$$

$\Delta \neq 0$ ekanligini ko‘rsatamiz, buning uchun quyidagi lemmani isbotlaymiz.

Lemma. Quyidagi

$$\begin{cases} X''' + k_n^3 X = 0, \\ X''(0) = X(p) = X'(p) = 0, \end{cases} \tag{19}$$

(19) chegaraviy masala faqat trivial $X(x) \equiv 0$ yechimga ega.

Isbot. $X(x) \neq 0$ deb faraz qilaylik. Quyidagi tenglikni ko‘rib chiqaylik

$$X\left(X''' + k_n^3 X \right) = 0.$$

Ushbu ifodani quyidagicha shakllantiramiz:

$$\left(XX'' - \frac{1}{2}(X')^2 \right)' + k_n^3 X^2 = 0,$$

$(0 < x < p)$ oraliqda integrallab, chegaraviy shartlarni inobatga olsak quyidagini olamiz:

$$\frac{1}{2}(X'(0))^2 + k_n^3 \int_0^p X^2 dx = 0.$$

Bu yerda $k_n^3 > 0$ bo‘lganligi sababli $X(x) \equiv 0$ ekanligi kelib chiqadi. **Lemma isbotlandi.**

Faraz qilaylik, $\Delta = 0$. Unda C_1^*, C_2^*, C_3^* doimiylari mavjud bo‘ladi hamda ulardan kamida bittasi bir vaqtning o‘zida nolga teng emas va sistema shartlariga javob beradi

$$\begin{cases} k_n^2\left(C_1^*-\frac{1}{2}C_2^*+\frac{\sqrt{3}}{2}C_3^*\right)=0, \\ C_1^*e^{-k_np}+e^{\frac{1}{2}k_np}\left(C_2^*\cos\frac{\sqrt{3}}{2}k_np+C_3^*\sin\frac{\sqrt{3}}{2}k_np\right)=0, \\ -k_nC_1^*e^{-k_np}+k_ne^{\frac{1}{2}k_np}\left(C_2^*\cos\left(\frac{\sqrt{3}}{2}k_np+\frac{\pi}{3}\right)+C_3^*\sin\left(\frac{\sqrt{3}}{2}k_np+\frac{\pi}{3}\right)\right)=0. \end{cases}$$

Shundan kelib chiqadiki,

$$V_n(x)=C_1^*e^{-k_nx}+C_2^*e^{\frac{1}{2}k_nx}\cos\frac{\sqrt{3}}{2}k_nx+C_3^*e^{\frac{1}{2}k_nx}\sin\frac{\sqrt{3}}{2}k_nx$$

funksiya (19) chegaraviy masalaning yechimi hisoblanadi. Lekin yuqorida isbotlangan lemmaga ko‘ra ushbu masalaning faqat trivial yechimi mavjud, ya’ni:

$$C_1^*e^{-k_nx}+C_2^*e^{\frac{1}{2}k_nx}\cos\frac{\sqrt{3}}{2}k_nx+C_3^*e^{\frac{1}{2}k_nx}\sin\frac{\sqrt{3}}{2}k_nx\equiv 0$$

bo‘lishi kerak edi. Lekin e^{-k_nx}, $e^{\frac{1}{2}k_nx}$, $\cos\frac{\sqrt{3}}{2}k_nx$, $\sin\frac{\sqrt{3}}{2}k_nx$ funksiyalarning chiziqli erkliligi tufayli, bu tenglik $C_1^*=C_2^*=C_3^*=0$ bo‘lgandagina o‘rinlidir. Bu esa bizning dastlabki (kamida bitta doimiy noldan farqli) farazimizga zid keladi. Olingan ziddiyat tufayli(16) tenglamalar sistemasining bosh determinanti hech qachon nolga teng bo‘la olmaydi, ya’ni $\Delta\neq 0$.

Bundan kelib chiqadiki (16) tenglamalar sisitemasining yagona yechimi mavjud:

$$C_{1n}=\frac{1}{\Delta}\left[A_{1n}k_ne^{k_np}(\alpha\delta-\theta\gamma)+A_{2n}k_n^3e^{\frac{1}{2}k_np}\left(\frac{1}{2}\delta+\frac{\sqrt{3}}{2}\gamma\right)-A_{3n}k_n^2e^{\frac{1}{2}k_np}\left(\frac{1}{2}\theta+\frac{\sqrt{3}}{2}\alpha\right)\right],$$

$$C_{2n}=\frac{1}{\Delta}\left[-A_{1n}k_ne^{-\frac{1}{2}k_np}(\delta+\theta)+A_{2n}k_n^3\left(\delta e^{\frac{1}{2}k_np}+\frac{\sqrt{3}}{2}e^{-k_np}\right)+A_{3n}k_n^2\left(\frac{\sqrt{3}}{2}e^{-k_np}-\theta e^{\frac{1}{2}k_np}\right)\right],$$

$$C_{3n}=\frac{1}{\Delta}\left[A_{1n}k_ne^{-\frac{1}{2}k_np}(\alpha+\gamma)+A_{2n}k_n^3\left(\frac{1}{2}e^{-k_np}-\gamma e^{\frac{1}{2}k_np}\right)+A_{3n}k_n^2\left(\alpha e^{\frac{1}{2}k_np}+\frac{1}{2}e^{-k_np}\right)\right].$$

C_{in} larni olib borib (15) ga qo‘ysak quyidagini olamiz:

$$u(x,y)=\sum_{n=1}^{\infty}\left[A_{1n}B_{1n}(x)+A_{2n}B_{2n}(x)+A_{3n}B_{3n}(x)\right]\cdot y^{\frac{1}{2}-\beta}J_{\frac{1}{2}-\beta}\left(\frac{\sigma_ny}{q}\right) \qquad (20)$$

bu erda

$$B_{1n}(x)=\frac{\sqrt{3}k_n}{\Delta}\left[\frac{1}{2}e^{k_n(p-x)}-e^{-\frac{1}{2}k_n(p-x)}\sin\left(\frac{\sqrt{3}}{2}k_n(p-x)+\frac{\pi}{6}\right)\right],$$

$$B_{2n}(x)=\frac{k_n^3}{\Delta}\left[e^{k_n\left(\frac{1}{2}p-x\right)}\cos\left(\frac{\sqrt{3}}{2}k_np+\frac{\pi}{6}\right)+\right.$$

$$\left.+e^{\frac{1}{2}k_n(x+p)}\sin\left(\frac{\sqrt{3}}{2}k_n(p-x)+\frac{\pi}{3}\right)+e^{-\frac{1}{2}k_n(2p-x)}\cos\left(\frac{\sqrt{3}}{2}k_nx-\frac{\pi}{6}\right)\right],$$

$$B_{3n}(x)=\frac{k_n^2}{\Delta}\left[-e^{k_n\left(\frac{1}{2}p-x\right)}\cos\left(\frac{\sqrt{3}}{2}k_np-\frac{\pi}{6}\right)-\right.$$

$$\left.-e^{\frac{1}{2}k_n(p+x)}\sin\left(\frac{\sqrt{3}}{2}k_n(p-x)\right)+e^{-\frac{1}{2}k_n(2p-x)}\cos\left(\frac{\sqrt{3}}{2}k_nx-\frac{\pi}{6}\right)\right].$$

Agar (20) qator va uning $\frac{\partial^3u}{\partial x^3}$, $\frac{\partial^2u}{\partial y^2}$, $\frac{\partial u}{\partial y}$ hosilalari D sohada bir xilda yaqinlashsa, u holda (20) qator orqali aniqlanadigan $u(x,y)$ funksiya A_2 masalaning yechimini beradi.

Ushbu qatorni absolyut yaqinlashishini isbotlaymiz. (20) dan quyidagini olamiz

$$|u(x,y)|=\sum_{n=1}^{\infty}\left[\left|A_{1n}B_{1n}(x)+A_{2n}B_{2n}(x)+A_{3n}B_{3n}(x)\right|\right]\cdot\left|y^{\frac{1}{2}-\beta}J_{\frac{1}{2}-\beta}\left(\frac{\sigma_ny}{q}\right)\right|=$$

$$=\sum_{n=1}^{\infty}\left[\left|A_{1n}B_{1n}(x)+A_{2n}B_{2n}(x)+A_{3n}B_{3n}(x)\right|\right]\cdot\left|\frac{1}{\Gamma\left(\frac{3}{2}-\beta\right)2^{\frac{1}{2}-\beta}}\left(\frac{\sigma_n}{q}\right)^{\frac{1}{2}-\beta}y^{1-2\beta}\overline{J}_{\frac{1}{2}-\beta}\left(\frac{\sigma_ny}{q}\right)\right|\le$$

$$\le\sum_{n=1}^{\infty}\left[\left|A_{1n}B_{1n}(x)+A_{2n}B_{2n}(x)+A_{3n}B_{3n}(x)\right|\right]\cdot\left|\frac{q^{\frac{1}{2}-\beta}}{\Gamma\left(\frac{3}{2}-\beta\right)2^{\frac{1}{2}-\beta}}(\sigma_n)^{\frac{1}{2}-\beta}\right|$$

$$|u(x,y)|\le\sum_{n=1}^{\infty}|A_{1n}||B_{1n}(x)|\left|C_0(\sigma_n)^{\frac{1}{2}-\beta}\right|+\sum_{n=1}^{\infty}|A_{2n}||B_{2n}(x)|\left|C_0(\sigma_n)^{\frac{1}{2}-\beta}\right|+$$

$$+\sum_{n=1}^{\infty}|A_{3n}||B_{3n}(x)|\left|C_0(\sigma_n)^{\frac{1}{2}-\beta}\right| \qquad (21)$$

Bu yerda $C_0=\frac{q^{\frac{1}{2}-\beta}}{\Gamma\left(\frac{3}{2}-\beta\right)2^{\frac{1}{2}-\beta}}$.

Quyida A_{in} larni baholaymiz. Buning uchun bo‘laklab integrallashni amalga oshiramiz va (5) shartlardan foydalanamiz:

$$|A_{in}| = \left| \frac{\sqrt{2}}{qJ_{\frac{3}{2}-\beta}(\sigma_n)} \int_0^q \psi_i(\eta)\eta^{\frac{1}{2}-\beta} J_{\frac{1}{2}-\beta}\left(\frac{\sigma_n \eta}{q}\right) d\eta \right| =$$

$$= \left| \frac{-C_1}{J_{\frac{3}{2}-\beta}(\sigma_n)} \cdot \frac{q}{\sigma_n} \int_0^q \psi_i(\eta)\eta^{-2\beta} d\eta^{\frac{1}{2}+\beta} J_{-\frac{1}{2}-\beta}\left(\frac{\sigma_n \eta}{q}\right) \right|$$

Bu yerda $C_1 = \frac{\sqrt{2}}{q}$.

Birinchi marta bo‘laklab integrallaymiz:

$$|A_{in}| = \left| \frac{-C_1 q}{\sigma_n J_{\frac{3}{2}-\beta}(\sigma_n)} \left[\psi_i(\eta)\eta^{\frac{1}{2}-\beta} J_{-\frac{1}{2}-\beta}\left(\frac{\sigma_n \eta}{q}\right) \right|_{\eta=0}^{\eta=q} - \right.$$

$$\left. - \int_0^q \frac{d}{d\eta}\left(\psi_i(\eta)\eta^{-2\beta}\right)\eta^{\frac{1}{2}+\beta} J_{-\frac{1}{2}-\beta}\left(\frac{\sigma_n \eta}{q}\right) d\eta \right] \Bigg| =$$

$$= \left| \frac{C_1 q}{\sigma_n J_{\frac{3}{2}-\beta}(\sigma_n)} \int_0^q \frac{d}{d\eta}\left(\psi_i(\eta)\eta^{-2\beta}\right)\eta^{\frac{1}{2}+\beta} J_{-\frac{1}{2}-\beta}\left(\frac{\sigma_n \eta}{q}\right) d\eta \right|$$

Ikkinchi marta bo‘laklab integrallaymiz:

$$|A_{in}| = \left| \frac{C_1 q}{\sigma_n J_{\frac{3}{2}-\beta}(\sigma_n)} \int_0^q \eta^{2\beta} \frac{d}{d\eta}\left(\psi_i(\eta)\eta^{-2\beta}\right) d\eta^{\frac{1}{2}-\beta} J_{\frac{1}{2}-\beta}\left(\frac{\sigma_n \eta}{q}\right) \right| =$$

$$= \left| \frac{C_1 q^2}{\sigma_n{}^2 J_{\frac{3}{2}-\beta}(\sigma_n)} \eta^{\frac{1}{2}+\beta} \frac{d}{d\eta}\left(\psi_i(\eta)\eta^{-2\beta}\right) J_{\frac{1}{2}-\beta}\left(\frac{\sigma_n \eta}{q}\right) \right|_{\eta=0}^{\eta=q} +$$

$$+ \left| \frac{-C_1 q^2}{\sigma_n{}^2 J_{\frac{3}{2}-\beta}(\sigma_n)} \int_0^q \frac{d}{d\eta}\left(\eta^{2\beta} \frac{d}{d\eta}\left(\psi_i(\eta)\eta^{-2\beta}\right)\right)\eta^{\frac{1}{2}-\beta} J_{\frac{1}{2}-\beta}\left(\frac{\sigma_n \eta}{q}\right) d\eta \right|$$

Uchinchi marta bo‘laklab integrallaymiz:

$$|A_{in}| = \left| \frac{-C_1 q^2}{\sigma_n{}^2 J_{\frac{3}{2}-\beta}(\sigma_n)} \int_0^q \frac{d}{d\eta}\left(\eta^{2\beta} \frac{d}{d\eta}\left(\psi_i(\eta)\eta^{-2\beta}\right)\right)\eta^{\frac{1}{2}-\beta} J_{\frac{1}{2}-\beta}\left(\frac{\sigma_n \eta}{q}\right) d\eta \right| =$$

$$=\left|\frac{C_1 q^3}{\sigma_n{}^3 J_{\frac{3}{2}-\beta}(\sigma_n)}\int_0^q \eta^{-2\beta}\frac{d}{d\eta}\left(\eta^{2\beta}\frac{d}{d\eta}\left(\psi_i(\eta)\eta^{-2\beta}\right)\right)d\eta^{\frac{1}{2}+\beta}J_{-\frac{1}{2}-\beta}\left(\frac{\sigma_n\eta}{q}\right)\right|=$$

$$=\left|\frac{C_1 q^3}{\sigma_n{}^3 J_{\frac{3}{2}-\beta}(\sigma_n)}\frac{d}{d\eta}\left(\eta^{2\beta}\frac{d}{d\eta}\left(\psi_i(\eta)\eta^{-2\beta}\right)\right)\eta^{\frac{1}{2}-\beta}J_{-\frac{1}{2}-\beta}\left(\frac{\sigma_n\eta}{q}\right)\right|_{\eta=0}^{\eta=q}+$$

$$+\left|\frac{-C_1 q^3}{\sigma_n{}^3 J_{\frac{3}{2}-\beta}(\sigma_n)}\int_0^q \frac{d}{d\eta}\left[\eta^{-2\beta}\frac{d}{d\eta}\left(\eta^{2\beta}\frac{d}{d\eta}\left(\psi_i(\eta)\eta^{-2\beta}\right)\right)\right]\eta^{\frac{1}{2}+\beta}J_{-\frac{1}{2}-\beta}\left(\frac{\sigma_n\eta}{q}\right)d\eta\right|=$$

$$=\left|\frac{-C_1 q^3}{\sigma_n{}^3 J_{\frac{3}{2}-\beta}(\sigma_n)}\int_0^q \frac{d}{d\eta}\left[\eta^{-2\beta}\frac{d}{d\eta}\left(\eta^{2\beta}\frac{d}{d\eta}\left(\psi_i(\eta)\eta^{-2\beta}\right)\right)\right]\eta^{\frac{1}{2}+\beta}J_{-\frac{1}{2}-\beta}\left(\frac{\sigma_n\eta}{q}\right)d\eta\right|$$

To‘rtinchi marta bo‘laklab integrallaymiz:

$$|A_{in}|=\left|\frac{C_1 q^4}{\sigma_n{}^4 J_{\frac{3}{2}-\beta}(\sigma_n)}\cdot\right.$$

$$\left.\cdot\int_0^q \frac{d}{d\eta}\left[\eta^{2\beta}\frac{d}{d\eta}\left[\eta^{-2\beta}\frac{d}{d\eta}\left(\eta^{2\beta}\frac{d}{d\eta}\left(\psi_i(\eta)\eta^{-2\beta}\right)\right)\right]\right]\eta^{\frac{1}{2}-\beta}J_{\frac{1}{2}-\beta}\left(\frac{\sigma_n\eta}{q}\right)d\eta\right|$$

$$|A_{in}|=\left|\frac{C_1 q^4}{\sigma_n{}^4 J_{\frac{3}{2}-\beta}(\sigma_n)}\int_0^q \frac{d}{d\eta}\left\{\eta^{2\beta}\frac{d}{d\eta}\left[\eta^{-2\beta}\frac{d}{d\eta}\right.\right.\right.$$

$$\left.\left.\left.\left(\eta^{2\beta}\frac{d}{d\eta}\left(\psi_i(\eta)\eta^{-2\beta}\right)\right)\right]\right\}\eta^{\frac{1}{2}-\beta}J_{\frac{1}{2}-\beta}\left(\frac{\sigma_n\eta}{q}\right)d\eta\right|$$

Bu yerda quyidagicha belgilash kiritamiz:

$$\Psi_{in}=\frac{1}{J_{\frac{3}{2}-\beta}(\sigma_n)}\int_0^q \frac{d}{d\eta}\left\{\eta^{2\beta}\frac{d}{d\eta}\left[\eta^{-2\beta}\frac{d}{d\eta}\left(\eta^{2\beta}\frac{d}{d\eta}\left(\psi_i(\eta)\eta^{-2\beta}\right)\right)\right]\right\}\eta^{\frac{1}{2}-\beta}J_{\frac{1}{2}-\beta}\left(\frac{\sigma_n\eta}{q}\right)d\eta$$

Bulardan

$$|A_{in}|\le\left|C_1\frac{q^4}{\sigma_n{}^4}\Psi_{in}\right|,\quad i=1,2,3 \tag{22}$$

Quyida topilgan barcha ma’lum musbat doimiylarniz M bilan belgilaymiz. (22) ni hisobga olgan holda (20) ning baholashi quyidagi ko‘rinishda bo‘ladi:

$$|u(x,y)| \le M\left(\sum_{n=1}^{\infty}\frac{|\Psi_{1n}|}{n^{\frac{7}{2}+\beta}}\cdot|B_{1n}(x)|+\sum_{n=1}^{\infty}\frac{|\Psi_{2n}|}{n^{\frac{7}{2}+\beta}}\cdot|B_{2n}(x)|+\sum_{n=1}^{\infty}\frac{|\Psi_{3n}|}{n^{\frac{7}{2}+\beta}}|B_{3n}(x)|\right) \qquad (23)$$

Quyida $B_{in}(x)$ funksiyalarni baholaymiz:

$$|B_{1n}(x)| = \frac{\sqrt{3k_n}}{|\overline{\Delta}|}\left|\left[\frac{1}{2}e^{k_n(p-x)} - e^{-\frac{1}{2}k_n(p-x)}\sin\left(\frac{\sqrt{3}}{2}k_n(p-x)+\frac{\pi}{6}\right)\right]\right| \le \frac{M}{k_n^2},$$

$$|B_{2n}(x)| = \frac{k_n^3}{|\overline{\Delta}|}\left|\left[e^{k_n\left(\frac{1}{2}p-x\right)}\cos\left(\frac{\sqrt{3}}{2}k_np+\frac{\pi}{6}\right)+\right.\right.$$

$$\left.\left.+e^{\frac{1}{2}k_n(x+p)}\sin\left(\frac{\sqrt{3}}{2}k_n(p-x)+\frac{\pi}{3}\right)+e^{-\frac{1}{2}k_n(2p-x)}\cos\left(\frac{\sqrt{3}}{2}k_nx-\frac{\pi}{6}\right)\right]\right| \le M,$$

$$|B_{3n}(x)| = \frac{k_n^2}{|\overline{\Delta}|}\left|\left[-e^{k_n\left(\frac{1}{2}p-x\right)}\cos\left(\frac{\sqrt{3}}{2}k_np-\frac{\pi}{6}\right)-\right.\right.$$

$$\left.\left.-e^{\frac{1}{2}k_n(p+x)}\sin\left(\frac{\sqrt{3}}{2}k_n(p-x)\right)+e^{-\frac{1}{2}k_n(2p-x)}\cos\left(\frac{\sqrt{3}}{2}k_nx-\frac{\pi}{6}\right)\right]\right| \le \frac{M}{k_n}.$$

Bu yerda

$$\overline{\Delta} == \sqrt{3}k_n^3\left[\frac{1}{2}e^{k_np}+e^{-\frac{1}{2}k_np}\cos\left(\frac{\sqrt{3}}{2}k_np\right)\right].$$

Yuqoridagilarni hisobga olgan holda (23) dan quyidagini olamiz:

$$|u(x,y)| \le M\left(\sum_{n=1}^{\infty}\frac{|\Psi_{1n}|}{n^{\frac{11}{2}+\beta}}+\sum_{n=1}^{\infty}\frac{|\Psi_{2n}|}{n^{\frac{7}{2}+\beta}}+\sum_{n=1}^{\infty}\frac{|\Psi_{3n}|}{n^{\frac{9}{2}+\beta}}\right) < \infty$$

Bundan kelib chiqadiki (20) qator $\overline{D}$ sohada tekis yaqinlashadi.

Endi (20) qatorni hosilalarini ikkala o‘zgaruvchi bo‘yicha ham $\overline{D}$ sohada absolyut va tekis yaqinlashishini isbotlaylik.

Quyidagi tengliklarning mavjudligi kelib chiqadi:

$$B_{1n}^{(r)}(x) = \frac{\sqrt{3}k_n^{\,r+1}}{\Delta}\left((-1)^r\frac{1}{2}e^{k_n(p-x)} - e^{-\frac{1}{2}k_n(p-x)}\sin\left(\omega - r\frac{\pi}{6}\right)\right),$$

$$B_{2n}^{(r)}(x) = \frac{k_n^{\,r+3}}{\Delta}\left[(-1)^r e^{\frac{1}{2}k_n(2p-x)}\cos\left(\frac{\sqrt{3}}{2}k_np+\frac{\pi}{3}\right)+\right.$$

$$\left.+e^{\frac{1}{2}k_n(p+x)}\sin\left(\frac{\sqrt{3}}{2}k_n(p-x)+\frac{\pi}{3}-r\frac{\pi}{6}\right)+e^{-\frac{1}{2}k_n(2p-x)}\cos\left(\frac{\sqrt{3}}{2}k_nx-\frac{\pi}{6}+r\frac{\pi}{6}\right)\right],$$

$$B_{3n}^{(r)}(x)=\frac{k_n^{r+2}}{\Delta}\left[(-1)^{r+1}e^{\frac{1}{2}k_n(2p-x)}\cos\left(\frac{\sqrt{3}}{2}k_np-\frac{\pi}{6}\right)-\right.$$

$$\left.-e^{\frac{1}{2}k_n(p+x)}\sin\left(\frac{\sqrt{3}}{2}k_n(p-x)-r\frac{\pi}{6}\right)+e^{-\frac{1}{2}k_n(2p-x)}\cos\left(\frac{\sqrt{3}}{2}k_nx-\frac{\pi}{6}+r\frac{\pi}{6}\right)\right].$$

Bu yerda $\omega=\frac{\sqrt{3}}{2}k_n(p-x)+\frac{\pi}{6}$.

$B_{in}^{(r)}(x)$ funksiya uchun quyidagi baholashlar amal qiladi:

$$\left|B_{1n}^{(r)}\right|\le Mk_n^{r-2},$$
$$\left|B_{2n}^{(r)}\right|\le Mk_n^{r},$$
$$\left|B_{3n}^{(r)}\right|\le Mk_n^{r-1}.$$

Bu yerda $0<x<p$ va $r=0,1,2,3$.

x bo‘yicha hosilalarni hisoblab (20) dan quyidagini olamiz:

$$\frac{\partial^3u}{\partial x^3}=\sum_{n=1}^{\infty}\left[A_{1n}B_{1n}'''(x)+A_{2n}B_{2n}'''(x)+A_{3n}B_{3n}'''(x)\right]\cdot y^{\frac{1}{2}-\beta}J_{\frac{1}{2}-\beta}\left(\frac{\sigma_ny}{q}\right)$$

$$\left|u(x,y)\right|\le\sum_{n=1}^{\infty}\left|A_{1n}\right|\left|B_{1n}'''(x)\right|\left|C_0(\sigma_n)^{\frac{1}{2}-\beta}\right|+\sum_{n=1}^{\infty}\left|A_{2n}\right|\left|B_{2n}'''(x)\right|\left|C_0(\sigma_n)^{\frac{1}{2}-\beta}\right|+$$

$$+\sum_{n=1}^{\infty}\left|A_{3n}\right|\left|B_{3n}'''(x)\right|\left|C_0(\sigma_n)^{\frac{1}{2}-\beta}\right|\le M\left[\sum_{n=1}^{\infty}\frac{\left|\Psi_{1n}\right|}{n^{\frac{5}{2}+\beta}}+\sum_{n=1}^{\infty}\frac{\left|\Psi_{2n}\right|}{n^{\frac{1}{2}+\beta}}+\sum_{n=1}^{\infty}\frac{\left|\Psi_{3n}\right|}{n^{\frac{3}{2}+\beta}}\right]$$

Koshi-Bunyakovskiy va Bessel tengsizliklaridan foydalangan holda quyidagi baholashni olamiz:

$$\left|\frac{\partial^3u}{\partial x^3}\right|=M\left(\sqrt{\sum_{n=1}^{\infty}\left|\Psi_{1n}\right|^2}\sqrt{\sum_{m=1}^{\infty}\frac{1}{n^{5+2\beta}}}+\sqrt{\sum_{n=1}^{\infty}\left|\Psi_{2n}\right|^2}\sqrt{\sum_{m=1}^{\infty}\frac{1}{n^{1+2\beta}}}+\sqrt{\sum_{n=1}^{\infty}\left|\Psi_{3n}\right|^2}\sqrt{\frac{1}{n^{3+2\beta}}}\right)\le$$

$$\le M\left(\sqrt{\left\|\psi_{1n}\right\|^2}+\sqrt{\left\|\psi_{2n}\right\|^2}+\sqrt{\left\|\psi_{3n}\right\|^2}\right)\le\infty$$

Bu yerda

$$\sum_{n=1}^{\infty}\left|\Psi_{in}\right|^2=\left\|\psi_n^{(4)}\right\|^2_{L_2(0,q)}=\cos nst>0,\quad i=1,2,3$$

Shunday qilib $\frac{\partial^3u}{\partial x^3}$ funksiyaga mos qator absolyut va tekis yaqinlashadi. Xuddi shunday $\frac{\partial^2u}{\partial y^2}$ qatorning absolyut va tekis yaqinlashishini isbot qilamiz.

$Y_n(y)$ funksiyadan kerakli hosilalarni olamiz:

$$Y_n'(y)=\frac{C_1}{J_{\frac{3}{2}-\beta}(\sigma_n)}\frac{d}{dy}\left[y^{\frac{1}{2}-\beta}J_{\frac{1}{2}-\beta}\left(\frac{\sigma_n y}{q}\right)\right]=\frac{C_1}{J_{\frac{3}{2}-\beta}(\sigma_n)}\frac{\sigma_n}{q}y^{\frac{1}{2}-\beta}J_{-\frac{1}{2}-\beta}\left(\frac{\sigma_n y}{q}\right)$$

$$Y_n''(y)=\frac{C_1}{J_{\frac{3}{2}-\beta}(\sigma_n)}\frac{d^2}{dy^2}\left[y^{\frac{1}{2}-\beta}J_{\frac{1}{2}-\beta}\left(\frac{\sigma_n y}{q}\right)\right]=\frac{C_1}{J_{\frac{3}{2}-\beta}(\sigma_n)}\frac{\sigma_n}{q}\frac{d}{dy}\left[y^{\frac{1}{2}-\beta}J_{-\frac{1}{2}-\beta}\left(\frac{\sigma_n y}{q}\right)\right]=$$

$$=\frac{C_1}{J_{\frac{3}{2}-\beta}(\sigma_n)}\frac{\sigma_n}{q}\left(\frac{1}{2}-\beta\right)y^{-\frac{1}{2}-\beta}J_{-\frac{1}{2}-\beta}\left(\frac{\sigma_n y}{q}\right)-\frac{C_1}{J_{\frac{3}{2}-\beta}(\sigma_n)}\frac{\sigma_n^2}{q^2}y^{\frac{1}{2}-\beta}J_{\frac{1}{2}-\beta}\left(\frac{\sigma_n y}{q}\right)=$$

$$=-\frac{C_1}{J_{\frac{3}{2}-\beta}(\sigma_n)}\left[\frac{\sigma_n(1-2\beta)}{2q}y^{-\frac{1}{2}-\beta}J_{-\frac{1}{2}-\beta}\left(\frac{\sigma_n y}{q}\right)+\frac{\sigma_n^2}{q^2}y^{\frac{1}{2}-\beta}J_{\frac{1}{2}-\beta}\left(\frac{\sigma_n y}{q}\right)\right]$$

y bo‘yicha ikkinchi tartibli hosilani hisoblab, quyidagiga ega bo‘lamiz:

$$\frac{\partial^2 u}{\partial y^2}=\sum_{n=1}^{\infty}\left[A_{1n}B_{1n}(x)+A_{2n}B_{2n}(x)+A_{3n}B_{3n}(x)\right]\cdot\frac{d^2}{dy^2}\left[y^{\frac{1}{2}-\beta}J_{\frac{1}{2}-\beta}\left(\frac{\sigma_n y}{q}\right)\right]$$

Ushbu ifodani modulini olib, yuqoridan baholaymiz:

$$\left|\frac{\partial^2 u}{\partial y^2}\right|\le\sum_{n=1}^{\infty}\left[|A_{1n}||B_{1n}(x)|+|A_{2n}||B_{2n}(x)|+|A_{3n}||B_{3n}(x)|\right]\cdot\left|\frac{d^2}{dy^2}\left[y^{\frac{1}{2}-\beta}J_{\frac{1}{2}-\beta}\left(\frac{\sigma_n y}{q}\right)\right]\right|$$

Bessel funksiyasining differensial tenglamasidan foydalanib, ikkinchi tartibli hosila modulini quyidagicha baholaymiz:

$$\left|\frac{d^2}{dy^2}\left[y^{\frac{1}{2}-\beta}J_{\frac{1}{2}-\beta}\left(\frac{\sigma_n y}{q}\right)\right]\right|\le M\cdot\sigma_n^2\cdot y^{\frac{1}{2}-\beta}$$

Biz yuqorida aniqlagan $|A_{in}|\le\frac{M|\Psi_{in}|}{n^4}$ hamda $|B_{in}(x)|$ baholashlarni inobatga oladigan bo‘lsakquyidagi yakuniy tengsizlik hosil bo‘ladi:

$$\left|\frac{\partial^2 u}{\partial y^2}\right|\le M\left[\sum_{n=1}^{\infty}\frac{|\Psi_{1n}|}{n^{\frac{9}{2}+\beta}}+\sum_{n=1}^{\infty}\frac{|\Psi_{2n}|}{n^{\frac{5}{2}+\beta}}+\sum_{n=1}^{\infty}\frac{|\Psi_{3n}|}{n^{\frac{7}{2}+\beta}}\right]$$

Shunday qilib, Veyershtrass alomatiga ko‘ra (20) yechim qatori va uning tarkibidagi barcha xususiy hosilalar qatorlar $\overline{D}$ sohada absolyut va tekis yaqinlashishi to‘liq isbotlandi.

Teorema isbotlandi.

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Omonova Shoxsanam Toxirjon qizi

29 РАЗРАБОТКА ИНТЕЛЛЕКТУАЛЬНОЙ СИСТЕМЫ ОЦЕНКИ КАЧЕСТВА ПОСАДКИ ОДЕЖДЫ НА ВИРТУАЛЬНОМ МАНЕКЕНЕ
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30 UCHINCHI TARTIBLI BITTA SINGULYAR KOEFFITSIYENTGA EGA BO‘LGAN DIFFERENSIAL TENGLAMA UCHUN CHEGARAVIY MASALA
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