TOR TEBRANISH TENGLAMASI UCHUN INTEGRAL SHARTLI MASALA

Jo‘rayeva Dilnavoz Umidjon qizi
Farg‘ona davlat universiteti, tayanch doktorant.

Annotatsiya
Ushbu maqolada tor tebranish tenglamasi uchun integral shartli nolokal masala tadqiq qilingan. Masala yechimi tor tebranish tenglamasi uchun Koshi masalasiga keltirish orqali topilgan va yordamchi differensial tenglamalardan foydalanilib umumiy yechim ko‘rinishi topilgan. Asosiy teoremalarda masalaning yechimi mavjudligi va yagonaligi isbotlangan. Natijada tor tebranish tenglamasi uchun yangi turdagi integral shartli masalaning aniq yechim formulasiga ega bo‘linadi.

Kalit so‘zlar. *Tor tebranish tenglamasi, Nolokal masala, Integral shart, Koshi masalasi, Yechimning mavjudligi, Yechimning yagonaligi, Kechikuvchi argumentli tenglama, Differensial tenglama.*

I. Masalaning qo‘yilishi

Ushbu $t-x=0$, $t+x=1$ va $t=0$ chiziqlar bilan chegaralagan soha D bilan belgilaylik. D sohada quyidagi tor tebranish tenglamasini qaraylik:

$$u_{tt}(x,t)-u_{xx}(x,t)=f(x,t). \tag{1}$$

Bu tenglama uchun quyidagi nolokal shartli masalani qaraymiz.

I_1 **masala.** D sohada (1) tenglamani qanoatlantiruvchi, $u,u_t\in C\left(\overline{D}\right)$, $u_{xx},u_{tt}\in C(D)$ sinfga tegishli va quyidagi

$$u(x/2,x/2)+\alpha\int_0^x u(1-\xi,0)\,d\xi=\varphi(x),\ 0\le x\le 1, \tag{2}$$

$$u_t(x,0)=v(x),\ 0\le x\le 1, \tag{3}$$

shartlarni qanoatlantiruvchi $u(x,t)$ funksiya topilsin, bu yerda $\varphi(x),v(x)$ -berilgan funksiyalar, α - berilgan noldan farqli haqiqiy son.

II. I_1 masalaning tadqiqoti

Ushbu masalani yechish uchun

$$u(x,0)=\tau(x),\ 0\le x\le 1, \tag{4}$$

belgilashni kiritaylik, bu yerda $\tau(x)$ - noma’lum funksiya.

Endi (1) tenglama uchun (3) va (4) shartlarni qanoatlantiruvchi Koshi masalasi hosil bo‘ldi

1-teorema. Agar $\tau(x)\in C^1[0,1]\cap C^2(0,1)$, $f(x,t)\in L(D)$ va $v(x)\in C[0,1]$ bo‘lsa, u holda Koshi masalasining yagona yechimi

$$u(x,t)=\frac{1}{2}\left[\tau(x-t)+\tau(x+t)\right]+\frac{1}{2}\int_{x-t}^{x+t}v(z)\,dz+\frac{1}{2}\int_0^t d\eta\int_{x-t+\eta}^{x+t-\eta}f(s,\eta)\,ds \tag{5}$$

ko‘rinishda bo‘ladi. [2]

Endi noma’lum $\tau(x)$ funksiyani topish uchun (5) yechimni (2) shartga bo‘ysundirib,

$$\frac{1}{2}\left[\tau(0)+\tau(x)\right]+\frac{1}{2}\int_0^x v(z)dz+\frac{1}{2}\int_0^{x/2} d\eta\int_\eta^{x-\eta} f(s,\eta)ds+\alpha\int_0^x \tau(1-\xi)d\xi=\varphi(x),\ 0\le x\le 1, \quad (6)$$

integral tenglamani hosil qilamiz.

(6) da $x=0$ deb $\tau(x)\in C[0,1]$ va $v(x)\in C[0,1]$ ekanligini e’tiborga olib,

$$\tau(0)=\varphi(0) \quad (7)$$

shartni hosil qilamiz va (6) ni quyidagicha yozib olamiz:

$$\tau(x)+2\alpha\int_0^x \tau(1-\xi)d\xi=F(x) \qquad 0\le x\le 1, \quad (8)$$

Bu yerda $F(x)$ning qiymati quyidagiga teng.

$$F(x)=\varphi(x)-\varphi(0)-\int_0^x v(z)dz-\int_0^{x/2} d\eta\int_\eta^{x-\eta} f(s,\eta)ds,\ 0\le x\le 1,$$

Endi (8) tenglamadan bir marta x bo‘yicha hosila olib,

$$\tau'(x)+2\alpha\tau(1-x)=F'(x), \quad (9)$$

kechikuvchi argumentli differensial tenglamani hosil qilamiz.

Agar (7) va (9) shartlarni qanoatlantiruvchi $\tau(x)$ funksiyani bir qiymatli topsak, I_1 masalani yechimini bir qiymatli topishimiz mumkin.

Yordamchi masalalardan foydalanib (7), (9) masalaning yechimini quyidagi ko‘rinishda topamiz:

$$\tau(x)=\frac{\varphi(0)}{\cos 2\alpha}\left(\cos(2\alpha-2\alpha x)-\sin(2\alpha x)\right)+$$

$$+\frac{\cos(2\alpha-2\alpha x)-\sin(2\alpha x)}{\cos 2\alpha}\int_0^1 \cos(2\alpha s)\left[2\varphi'(s)-v(s)-\int_0^{s/2} f(s-\eta,\eta)d\eta\right]ds-$$

$$-\int_x^1 \frac{\cos(2\alpha-2\alpha x)\cos(2\alpha s)-\sin(2\alpha-2\alpha s)\sin(2\alpha x)}{\cos 2\alpha}\left[2\varphi'(s)-v(s)-\int_0^{s/2} f(s-\eta,\eta)d\eta\right]ds+$$

$$+\int_{1-x}^1 \frac{\sin(2\alpha x)\cos(2\alpha s)-\cos(2\alpha-2\alpha x)\sin(2\alpha-2\alpha s)}{\cos 2\alpha}\left[2\varphi'(s)-v(s)-\int_0^{s/2} f(s-\eta,\eta)d\eta\right]ds. \quad (10)$$

(10) ni bo‘laklab integrallab quyidagi ko‘rinishda yozib olamiz

$$\tau(x)=2\varphi(x)+\frac{\cos(2\alpha-2\alpha x)-\sin(2\alpha x)}{\cos 2\alpha}\int_0^1\left[4\alpha\cdot\varphi(s)\sin(2\alpha s)\right]ds-$$

$$-\frac{\cos(2\alpha-2\alpha x)-\sin(2\alpha x)}{\cos 2\alpha}\int_0^1 \cos(2\alpha s)\left[v(s)+\int_0^{s/2} f(s-\eta,\eta)d\eta\right]ds-$$

$$-\int_x^1 \frac{\cos(2\alpha-2\alpha x)\sin(2\alpha s)-\cos(2\alpha-2\alpha s)\sin(2\alpha x)}{\cos 2\alpha}4\alpha\cdot\varphi(s)ds+$$

$$+\int_x^1 \frac{\cos(2\alpha-2\alpha x)\cos(2\alpha s)-\sin(2\alpha-2\alpha s)\sin(2\alpha x)}{\cos 2\alpha}\left[v(s)+\int_0^{s/2} f(s-\eta,\eta)d\eta\right]ds+ \quad (11)$$

$$+\int_{1-x}^{1}\frac{\sin(2\alpha x)\sin(2\alpha s)-\cos(2\alpha-2\alpha x)\cos(2\alpha-2\alpha s)}{\cos 2\alpha}4\alpha\cdot\varphi(s)\,ds-$$

$$-\int_{1-x}^{1}\frac{\sin(2\alpha x)\cos(2\alpha s)-\cos(2\alpha-2\alpha x)\sin(2\alpha-2\alpha s)}{\cos 2\alpha}\left[v(s)+\int_{0}^{s/2}f(s-\eta,\eta)d\eta\right]ds$$

(11) ni (5) ga qo‘yib, I_1 masalaning yechimi hosil qilamiz.

2-teorema. Agar $|\alpha|<\frac{1}{2}$, $\varphi(x)\in C^1[0,1]\cap C^2(0,1)$, $f(x,t)\in L(D)$ va $v(x)\in C[0,1]\cap C^1(0,1)$ bo‘lsa, u holda I_1 masalaning yechimi mavjud va yagona bo‘lib, u (5) va (11) formulalar bilan aniqlanadi.

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