A BOUNDARY-VALUE PROBLEM FOR A COUPLED SECOND KIND DEGENERATE HYPERBOLIC EQUATIONS

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Abstract

In the present paper, we formulate and study a boundary value problem for a system of coupled hyperbolic equations of the second kind in a domain consisting of the characteristics of the equations. Using the representation of the R_2-generalized solution of the equation and the properties of fractional differential and integral operators, the problem is equivalently reduced to a Volterra integral equation of the second kind with respect to the trace of the derivative of the solution.

Keywords: Degenerate hyperbolic equations, boundary-value problem, coupled system, Riemann function, Volterra integral equation, characteristic method.

INTRODUCTION

Formulation of the problem. We consider the equation

$$|y|^l u_{xx} - u_{yy} = 0, \tag{1.1}$$

where $l = n_1$ for $y > 0$ and $l = n_2$ for $y < 0$ with $-1 < l < 0$, in a bounded domain Q enclosed by the characteristics

$$AC: x - \frac{2}{n_1+2} y^{\frac{n_1+2}{2}} = 0, \qquad BC: x + \frac{2}{n_1+2} y^{\frac{n_1+2}{2}} = 1,$$

$$AD: x - \frac{2}{n_2+2}(-y)^{\frac{n_2+2}{2}} = 0, \quad BD: x + \frac{2}{n_2+2}(-y)^{\frac{n_2+2}{2}} = 1,$$

corresponding to equation (1.1).

Let us introduce the following notations: $\Omega^+ = \Omega \cap \{y > 0\}$ and $\Omega^- = \Omega \cap \{y < 0\}$. Moreover, by I we denote the interval $0 < x < 1$ on the line $y = 0$. Also,

$$u(x,y) = \begin{cases} u^+(x,y), & (x,y) \in \Omega^+, \\ u^-(x,y), & (x,y) \in \Omega^-. \end{cases}$$

It is known that the solution of the Cauchy problem for equation (1.1) in the domain Ω^+ (respectively, Ω^-) has the form [1]

$$u^{\pm}(x,y) = \frac{\Gamma(2+2\beta_i)}{\Gamma^2(1+\beta_i)} \int_0^1 \tau_{\pm}(t_i) \zeta^{\beta_i} (1-\zeta)^{\beta_i} d\zeta +$$

$$+ \frac{\Gamma(2+2\beta_i)}{(n_i+2)\Gamma^2(1+\beta_i)} (\pm y)^{\frac{n_i+2}{2}} \int_0^1 \tau'_{\pm}(t_i) \zeta^{\beta_i} (1-\zeta)^{\beta_i} (2\zeta - 1) d\zeta +$$

$$+ \frac{\Gamma(2-2\beta_i)}{\Gamma^2(1-\beta_i)} y \int_0^1 \nu_{\pm}(t_i) \zeta^{-\beta_i} (1-\zeta)^{-\beta_i} d\zeta, \tag{1.2}$$

where

$$\tau_{\pm}(x)=\lim_{y\to\pm 0}u^{\pm}(x,y),\qquad \nu_{\pm}(x)=\lim_{y\to\pm 0}u_y^{\pm}(x,y),\tag{1.3}$$

$$t_i=x+\frac{2}{n_i+2}(\pm y)^{(n_i+2)/2}(2\zeta-1),\quad \beta_i=\frac{n_i}{2(n_i+2)},\quad i=1,2.$$

We shall assume that the function $u^{+}(x,y)$ (respectively, $u^{-}(x,y)$) is an R_2-generalized solution of the Cauchy problem for equation (1.1) [1] if it can be represented in the form (1.2), where

$$\tau_{+}(x)=\tau_{+}(0)+\int_0^x(x-t)^{-2\beta_1}T_{+}(t)dt,\qquad \tau_{-}(x)=\tau_{-}(1)+\int_0^x(t-x)^{-2\beta_2}T_{-}(t)dt,$$

and the functions $T_{+}(t)$, $T_{-}(t)$, as well as $\nu_{+}(t)$, $\nu_{-}(t)$ are continuous and integrable on $(0,1)$.

We will study the following problem for equation (1.1):

Problem 1. Find a solution of equation (1.1) in the class R_2 satisfying the boundary conditions

$$u^{+}\big|_{AC}=\psi_{+}(x),\quad 0\le x\le\frac{1}{2};\tag{1.4}$$

$$u^{-}\big|_{AD}=\psi_{-}(x),\ 0\le x\le\frac{1}{2}\tag{1.5}$$

and the gluing (transmission) conditions

$$\nu_{+}(x)=\nu_{-}(x),\ x\in I;\tag{1.6}$$

$$\tau_{+}(x)=\tau_{-}(x),\ x\in\overline{I}\tag{1.7}$$

Where $\psi_{+}(x),\psi_{-}(x)\in C^1(\overline{I})\cap C^2(I)$, $\psi_{+}''(x)$ and $\psi_{-}''(x)$ may have singularities of order less than one at $x=0$ and $x=1$, respectively.

In the class R_2, on the basis of (1.2), from the boundary (1.3) and (1.4) we obtain [2]

$$\tau_{+}(x)=\chi_3^{+}\int_0^x(x-t)^{-2\beta_1}\nu(t)dt+\Phi_1^{+}(x),\tag{1.8}$$

$$\tau_{-}(x)=\chi_3^{-}\int_0^x(x-t)^{-2\beta_2}\nu(t)dt+\Phi_1^{-}(x),\tag{1.9}$$

where $\chi_3^{+}=2\chi_2^{+}\cos\pi\beta_1$, $\chi_3^{-}=2\chi_2^{-}\cos\pi\beta_2$

$$\chi_2^{+}=-\left[2(1-2\beta_1)\right]^{2\beta_1}\frac{\Gamma(-2\beta_1)}{\beta_1\Gamma^2(-\beta_1)},\ \chi_2^{-}=-\left[2(1-2\beta_2)\right]^{2\beta_2}\frac{\Gamma(-2\beta_2)}{\beta_2\Gamma^2(-\beta_2)},$$

$$\Phi_1^{+}(x)=\frac{2\Gamma(1+\beta_1)}{\Gamma(1+2\beta_1)}D_0^{2\beta_1-1}\left[x^{\beta_1}D_0^{1-\beta_1}\psi_{+}(x)\right],$$

$$\Phi_1^{-}(x)=\frac{2\Gamma(1+\beta_2)}{\Gamma(1+2\beta_2)}D_0^{2\beta_2-1}\left[x^{\beta_2}D_0^{1-\beta_2}\psi_{-}(x)\right].$$

Hence, considering the gluing condition (1.7), from the last equalities, we obtain

$$\chi_3^+\int_0^x (x-t)^{-2\beta_1}\nu(t)dt-\chi_3^-\int_0^x (x-t)^{-2\beta_2}\nu(t)dt=\Phi_2(x), \qquad (1.10)$$

where

$\Phi_2(x)=\Phi_1^-(x)-\Phi_1^+(x)$

Using the definition of the Riemann-Liouville fractional integral operator, we rewrite (1.10) in the form

$$D_{0x}^{2\beta_1-1}\nu_+(x)=\chi_4\int_0^x (x-t)^{-2\beta_2}\nu(t)dt+\frac{1}{\chi_3^+\Gamma(1-2\beta_1)}\Phi_2(x),$$

where $\chi_4=\dfrac{\chi_3^-}{\chi_3^+\Gamma(1-2\beta_1)}$.

Hence, applying differential operator $D_{0x}^{1-2\beta_1}$, we obtain

$$D_{0x}^{1-2\beta_1}D_{0x}^{2\beta_1-1}\nu_+(x)=\chi_4 D_{0x}^{1-2\beta_1}\int_0^x (x-t)^{-2\beta_2}\nu_-(t)dt+\Phi_3(x),$$

where

$$\Phi_3(x)=\frac{1}{\chi_3^+\Gamma(1-2\beta_1)}D_{0x}^{1-2\beta_1}\Phi_2(x).$$

Using the representation of the Riemann-Liuvill fractional differential operator, after some computations, the function $\Phi_3(x)$ can be written as

$$\Phi_3(x)=\frac{2\Gamma(1+\beta_2)}{\chi_3\Gamma(1-\beta_1)\Gamma(1+2\beta_2)\Gamma(1+2\beta_1-\beta_2)}x^{\beta_2}\times$$

$$\times\int_0^x \psi_-''(t)(x-t)^{2\beta_1-\beta_2}F\left(-\beta_2,2\beta_1-2\beta_2,2\beta_1-\beta_2+1;\frac{x-t}{x}\right)dt-$$

$$-\frac{x^{\beta_1}}{\chi_3}\Gamma(1-2\beta_1)\Gamma(1+2\beta_1)\int_0^x \psi_-''(t)(x-t)^{\beta_1}dt.$$

Hence, we obtain

$$\nu_+(x)-\frac{\lambda}{\Gamma(2\beta_1-2\beta_2)}\int_0^x\frac{\nu_-(t)dt}{(x-t)^{1-2\beta_1-2\beta_2}}=\Phi_3(x), \qquad (1.11)$$

where $\lambda=\chi_4\Gamma(1-2\beta_2)$.

It is known that the solution of the following integral equation (1.11) has the following form [3]

$$\nu_+(x)=\frac{d}{dx}\int_0^x E_{2\beta_1-2\beta_2,1}\left[\lambda(x-t)^{2\beta_1-2\beta_2}\right]\Phi_3(t)dt,$$

Where $E_{\alpha,\beta}(z)$ is the Mittag-Leffler function defined by

$$E_{\alpha,\beta}(z)=\sum_{k=0}^{+\infty}\frac{z^k}{\Gamma(\alpha k+\beta)}.$$

Next, we will find the function $\tau_{\pm}(x)$ using the formula (1.9):

$$\tau_{\pm}(x)=\Gamma(1-2\beta_1)\chi_3\int_0^x(x-t)^{-2\beta_1}E_{2\beta_1-\beta_2,1-2\beta_1}\left[\lambda(x-t)^{2\beta_1-2\beta_2}\right]\Phi_3(t)+\Phi_1^+(x)$$

Substituting the expressions of the functions $\tau_{\pm}(x)$ and $\nu_{\pm}(x)$ into (1.2), we have the representation of the solution the considered problem.

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