THE INVERSE-SOURCE PROBLEM FOR MIXED-TYPE EQUATIONS

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Abstract

Mixed-type differential equations combining hyperbolic and parabolic parts arise naturally in many important processes of mathematical physics, including heat conduction, diffusion phenomena, fluid dynamics, and wave propagation in heterogeneous media. Such equations describe systems in which different physical regimes coexist in separate subdomains and are connected through transmitting conditions. A special place in the theory of mixed-type equations is occupied by problems with fractional-order derivatives, since fractional models allow a more accurate description of memory and hereditary properties of complex media. In recent years, considerable attention has been devoted to boundary value and inverse problems for fractional mixed-type equations.

The method of solution is based on spectral expansion with respect to the eigenfunctions of the associated Sturm–Liouville problem, reduction to systems of fractional ordinary differential equations, and the application of Mittag-Leffler functions. Explicit formulas for the Fourier coefficients of the solution and unknown functions are derived, and sufficient conditions for existence and uniqueness are established. The obtained results extend previous studies on inverse problems for mixed-type equations and contribute to the further development of fractional differential equation theory.

Keywords: *Wave-Diffusion equation, inverse-source problem, the Caputo fractional order derivatives, Mittag-Lefler function, transcendent equation, the Cauchy problem.*

FORMULATION OF A PROBLEM

Problem. We aim to find functions $u(t,x)\in C(\bar{\Omega})\cap C^2(\Omega_1\cup\Omega_2)$, $u(\cdot,x)\in AC^2[0,a)\cup AC^1[a,b)$, $h(x)\in C[0,1]$ which satisfies the equation:

$$H(a-t)\,{}^{C}D_{0t}^{\alpha}u(t,x)+H(t-a)\,{}^{C}D_{at}^{\beta}u(t,x)-u_{xx}(t,x)=h(x) \tag{1}$$

together with the initial condition:

$$u(0,x)=\varphi(x), \tag{2}$$

boundary conditions:

$$u(t,0)=0,\ u(t,1)=0, \tag{3}$$

the over-determination condition for fixed $\xi\in(a,b)$:

$$u(\xi,x)=\psi(x), \tag{4}$$

and the transmitting conditions:

$$\lim_{t\to a+0}{}^{C}D_{at}^{\beta}u(t,x)=u_t(t,x) \tag{5}$$

$$u(a+0,0)=u(a-0,0) \tag{6}$$

where $H(\cdot)$ is a Heaviside function, $\Omega=\Omega_1\cup\Omega_2\cup\{t=a\}$, $\Omega_1=\{(t,x):0<x<1,0<t<a\}$, $\Omega_2=\{(t,x):0<x<1,a<t<b\}$, ${}^{C}D_{0t}^{\alpha}u(t,x)$ and ${}^{C}D_{at}^{\beta}u(t,x)$ -represent the Caputo fractional order derivatives [1], $\varphi(x),\psi(x)$ are given functions and with $a,b,\alpha,\beta\in\square$ such that $b>a>0,1<\alpha<2,0<\beta<1$.

Theorem.

Let condition

$$\Delta_k=\lambda_k M_2 E_{\beta,1}(-\lambda_k(\xi-a)^{\beta})E_{\alpha,1}(-\lambda_k a^{\alpha})+\lambda_k{}^2 M_1 a E_{\alpha,2}(-\lambda_k a^{\alpha})+$$
$$+\lambda_k M_1 E_{\alpha,1}(-\lambda_k a^{\alpha})-\lambda_k M_3 E_{\beta,1}(-\lambda_k(\xi-a)^{\beta})aE_{\alpha,2}(-\lambda_k a^{\alpha})\neq 0$$

holds. If

$$\psi(x)\in C^2[0,1],\psi'''(x)\in L_2(0,1),\psi(0)=\psi(1)=0,\psi''(0)=\psi''(1)=0,$$
$$\varphi(x)\in C[0,1],\varphi'(x)\in L_2(0,1),\varphi(0)=\varphi(1)=0,$$

there exists the unique solution of the inverse source problem, represented by

$$u(t,x)=\begin{cases}\sum_{k=1}^{\infty}[\varphi_k E_{\alpha,1}(-\lambda_k t^{\alpha})+B_k tE_{\alpha,1}(-\lambda_k t^{\alpha})+\\ t^{\alpha}E_{\alpha,\alpha+1}(-\lambda_k t^{\alpha})h_k]\sin\pi kx,\ 1<\alpha<2,0<t<a,\\ \sum_{k=1}^{\infty}[C_k E_{\beta,1}(-\lambda_k(t-a)^{\beta})+\\ (t-a)^{\beta}E_{\beta,\beta+1}(-\lambda_k(t-a)^{\beta})h_k]\sin\pi kx,\ 1<\beta<2,a<t<b\end{cases}$$

and unknown $h(x)$ function will be found by

$$h(x)=\sum_{k=1}^{\infty}[\frac{\lambda_k(M_4E_{\beta,1}(-\lambda_k(\xi-a)^{\beta})-\psi_k)E_{\alpha,1}(-\lambda_k a^{\alpha})}{\Delta_k}+$$
$$\frac{\lambda_k(\lambda_k\psi_k-M_2E_{\beta,1}(-\lambda_k(\xi-a)^{\beta}))aE_{\alpha,2}(-\lambda_k a^{\alpha})}{\Delta_k}]\sin\pi kx,$$

where

$$M_1=(\xi-a)^{\beta}E_{\beta,\beta+1}(-\lambda_k(\xi-a)^{\beta}),$$
$$M_2=a^{\alpha}E_{\alpha,\alpha+1}(-\lambda_k a^{\alpha}),$$
$$M_3=a^{\alpha-1}E_{\alpha,\alpha}(-\lambda_k a^{\alpha})-1,$$
$$M_4=-\varphi_k E_{\alpha,1}(-\lambda_k a^{\alpha}),$$
$$M_5=\lambda_k\varphi_k a^{\alpha-1}E_{\alpha,\alpha}(-\lambda_k a^{\alpha}),$$
$$B_k=\frac{\lambda_k(M_2M_5-M_3M_4)E_{\beta,1}(-\lambda_k(\xi-a)^{\beta}-\lambda_k M_3\psi_k-\lambda_k{}^2M_2\psi_k+\lambda_k{}^2M_1M_4+\lambda_k M_1M_5}{\Delta_k},$$

$$C_k = \frac{\lambda_k(\psi_k M_2 - M_1 M_4)E_{\alpha,1}(-\lambda_k a^\alpha) + \lambda_k(M_1 M_5 - \psi_k M_3)aE_{\alpha,2}(-\lambda_k a^\alpha)}{\Delta_k}.$$

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