BUZILADIGAN YUQORI JUFT TARTIBLI XUSUSIY HOSILALI DIFFERENSIAL TENGLAMA UCHUN LOKAL SHARTLI BOSHLANG‘ICH-CHEGARAVIY MASALA

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Annotatsiya

Ushbu ishda to‘rtburchak sohada vaqt o‘zgaruvchisi bo‘yicha ham, fazo o‘zgaruvchisi bo‘yicha ham buziladigan, kichik tartibli hadga ega bo‘lgan yuqori juft tartibli xususiy hosilali differensial tenglama uchun boshlang‘ich-chegaraviy masala qo‘yilgan va o‘rganilgan. Masala yechimining yagonaligini isbotlash uchun energiya integrallari usulidan foydalanilgan. Spektral analiz va Grin funksiyasi usullaridan foydalangan holda o‘rganilayotgan masalaning yechimi Furye qatori ko‘rinishida qurilgan. Grin funksiyasining koeffitsiyentlari ixtiyoriy n uchun topilgan. Yechim uchun baho olingan, undan berilgan funksiyalarga uzluksiz bog‘liqlik kelib chiqadi.

Kalit so‘zlar: *buziladigan tenglama, vaqt bo‘yicha buzilish, boshlang‘ich-chegaraviy masala, o‘zgaruvchilarni ajratish usuli, spektral masala, Grin funksiyasi usuli, Furye qatori.*

KIRISH

Quruvchanlik mexanikasida muhim ahamiyatga ega bo‘lgan balka va plastinalarning tebranishlariga oid ko‘pgina masalalar juft tartibli differensial tenglamalarga olib keladi. Vaqt va fazo o‘zgaruvchilari bo‘yicha buzilishga ega tenglamalar bir jinsli bo‘lmagan muhitlardagi diffuziya jarayonlarini, chegara qatlami nazariyasini, matematik biologiya masalalarini modellashtirishda paydo bo‘ladi.

So‘nggi yillarda yuqori juft tartibli buziladigan xususiy hosilali differensial tenglamalar uchun boshlang‘ich-chegaraviy masalalar jadal o‘rganilmoqda

Ushbu ishning asosiy yangiligi - buziladigan yuqori juft tartibli xususiy hosilali differensial tenglama uchun yangi boshlang‘ich-chegaraviy masalani qo‘yish, mos spektral masalaning Grin funksiyasini ixtiyoriy n uchun oshkor ko‘rinishda qurish hamda yechimning mavjudligi, yagonaligi va turg‘unligini isbotlashdan iborat.

Masalaning qo‘yilishi. $\Omega=\{(x,t):0<x<1,0<t<T\}$ to‘rtburchak sohada quyidagi buziladigan tenglamani qaraymiz:

$$t^a(1-t)^b\frac{\partial u}{\partial t}+cu+\frac{\partial^{2n}}{\partial x^{2n}}\left(x^\alpha\frac{\partial^{2n}u}{\partial x^{2n}}\right)=f(x,t),\quad n\in\square, \tag{1}$$

bu yerda $0\le a<1$, $0\le b<1$, $c\ge 0$, $0<\alpha<1$ -berilgan haqiqiy sonlar.

Quyidagi shartlarni qanoatlantiruvchi $u(x,t)$ funksiya topilsin:

1) $u,\partial u/\partial x,\ldots,\partial^{2n-1}u/\partial x^{2n-1}\in C(\bar{\Omega})$; $t^a(1-t)^b\partial u/\partial t\in C(\Omega)$;

2) Ω sohada (1) tenglamani qanoatlantiradi;

3) quyidagi boshlang‘ich va chegaraviy shartlarni qanoatlantiradi:

$$u(x,0)=\varphi(x),x\in[0,1], \tag{2}$$

$$\frac{\partial^j u(0,t)}{\partial x^j}=0,\frac{\partial^j}{\partial x^j}\left(x^\alpha\frac{\partial^{2n}u}{\partial x^{2n}}\right)\Bigg|_{x=1}=0,\ j=\overline{0,2n-1},t\in[0,T]. \tag{3}$$

Spektral masala va Grin funksiyasi. (1)-(3) masalaga Furye o‘zgaruvchilarni ajratish usulini formal qo‘llab, quyidagi spektral masalani hosil qilamiz:

$$M[v] \equiv \left(x^{\alpha} v^{(2n)}(x)\right)^{(2n)} = \lambda v(x), 0 < x < 1, \tag{4}$$

$$v^{(j)}(0) = 0, \left(x^{\alpha} v^{(2n)}\right)^{(j)} \Big|_{x=1} = 0, j = \overline{0, 2n-1}. \tag{5}$$

(5) shartlarni qanoatlantiruvchi ixtiyoriy v, w uchun bo‘laklab integrallash M operatorining o‘z-o‘ziga qo‘shmaligini ko‘rsatadi. Energiya tengligidan $\lambda \geq 0$, $\lambda = 0$ esa faqat trivial yechim berishi kelib chiqadi. Demak barcha xos qiymatlar musbat: $\lambda_k > 0$. (4),(5) masalaning $G(x,s)$ Grin funksiyasi quyidagi ko‘rinishda aniqlanadi:

$$G(x,s) = \begin{cases} \sum_{j=1}^{2n} \dfrac{(-1)^j s^{j-1} x^{4n-\alpha-j}}{(j-1)!(2n-j)!(2n-\alpha-j+1)_{2n}}, 0 \leq x \leq s, \\ \sum_{j=1}^{2n} \dfrac{(-1)^{j+1} s^{2n+j-1-\alpha} x^{2n-j}}{(j-1)!(2n-j)!(j-\alpha)_{2n}}, s \leq x \leq 1, \end{cases} \tag{6}$$

bu yerda $(z)_m = z(z+1)\cdots(z+m-1)$ Pochhammer simvoli. chegaraviy shartlarning o‘ziga xosligi shundaki, koeffitsiyentlarning yarmi aynan nolga aylanadi: $a_{2n+j} = 0$, $b_j = 0$, $j = \overline{1, 2n}$. Bu sabab Grin funksiya (6) juda sodda faqat bir hadli yig‘indilar ko‘rinishini oladi.

Furye qatori ko‘rinishida yechim. Masala yechimini

$$u(x,t) = \sum_{k=1}^{+\infty} u_k(t) v_k(x) \tag{7}$$

ko‘rinishida izlaymiz. (7) ni (1)-(2) ga qo‘yib, har bir k uchun Koshi masalasini hosil qilamiz:

$$t^a (1-t)^b u_{k'}(t) + (c + \lambda_k) u_k(t) = f_k(t), \quad u_k(0) = \varphi_k,$$

bu yerda $\varphi_k = \int_0^1 \varphi(x) v_k(x) dx$, $f_k(t) = \int_0^1 f(x,t) v_k(x) dx$.

Integrallovchi ko‘paytuvchi $\mu_k(t) = \exp\left((c+\lambda_k) \int_0^t d\tau / [\tau^a (1-\tau)^b]\right)$ yordamida yagona yechim topiladi:

$$u_k(t) = \varphi_k \mu_k^{-1}(t) + \mu_k^{-1}(t) \int_0^t \frac{\mu_k(\tau) f_k(\tau)}{\tau^a (1-\tau)^b} d\tau. \tag{8}$$

1-teoema. $\varphi(x)$ funksiya tegishli silliqlik shartlarini va chegaraviy shartlarini qanoatlantirsin, $f(x,t)$ funksiya esa x bo‘yicha t ga nisbatan tekis ravishda mos integrallanuvchanlik shartlarini qanoatlantirsin. U holda koeffitsiyentlari (8) formula bilan berilgan (7) qator masalaning yechimini ifodalaydi; ushbu qator va uning x hamda t bo‘yicha kerakli hosilalari $\bar{\Omega}$ (mos kompaktlarda) tekis yaqinlashadi.

2-teorema. Masala bittadan ortiq yechimga ega bo‘lishi mumkin emas.

Isbot. Ikkita yechimning ayirmasi $u = u_1 - u_2$ masalani qanoatlantiradi. (1) tenglamani ($f \equiv 0$) u ga ko‘paytirib, Ω_{T_0} bo‘yicha integrallab, bo‘laklab integrallashdan chegaraviy shartlari hisobiga yo‘qoladi. Hosil bo‘lgan energiya tengligiga Gronuoll lemmasini qo‘llash natijasida, $u \equiv 0$ kelib chiqadi.

3-teorema. Teoremalardagi shartlar bajarilganda masalaning yechimi uchun:

$$\| u(\cdot,t) \|^2_{L_2(0,1)} \le K_0 \left[\| \varphi \|^2_{L_2} + \| f \|^2_{L_2(\Omega)} \right], \tag{9}$$

$$\| u \|_{C(\bar{\Omega})} \le K_1 \left[\| \varphi^{(2n)} \|_{L_{2,r}(0,1)} + \| f \|_{L_2(\Omega)} \right], \tag{10}$$

baholar o‘rinli, bu yerda $\| \cdot \|_{L_{2,r}} = \left[\int_0^1 x^\alpha (\cdot)^2 dx \right]^{1/2}$, $K_0, K_1 > 0$ - konstantalar. (9) va (10) baholar yechimning boshlang‘ich va o‘ng tomon ma'lumotlariga uzluksiz bog‘liqligini ta'minlaydi.

XULOSA

Ushbu ishda to‘rtburchak sohada $x = 0$ va $t = [0, T]$ chegaralarida buziladigan yuqori juft tartibli xususiy hosilali differensial tenglama uchun boshlang‘ich-chegaraviy masala qaraldi. Bu masalada $4n$ -tartibli buziladigan operator uchun yuqoridagi chegaraviy shartlar bilan birinchi marta o‘rganildi, mos spektral masalaning Grin funksiyasi ixtiyoriy n uchun oshkor formulalar (6) ko‘rinishida qurildi, masalaning o‘z-o‘ziga qo‘shmali ekanligi ko‘rsatildi, energiya integrali usuli yordamida yechimning yagonaligi isbotlandi, yechim Furye qatori (7)-(8) ko‘rinishida qurildi va L_2 hamda $C(\bar{\Omega})$ fazolarda turg‘unlik baholari olindi.

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Муродова Мухлиса

10 BUZILADIGAN YUQORI JUFT TARTIBLI XUSUSIY HOSILALI DIFFERENSIAL TENGLAMA UCHUN LOKAL SHARTLI BOSHLANG‘ICH-CHEGARAVIY MASALA
Tolipov Nodirjon Isaqovich **49**

11 SiGe VA SiSn QATTIQ QORISHMALARI FIZIK XOSSALARINING SHAKLLANISHIDA TARKIB VA MEXANIK ZO‘RIQISHNING AHAMIYATI
Madaminov Xurshidjon Muxamedovich
Abdurahmonov Azizbek Abdurahim o‘g‘li **52**

12 Ce VA Yb BILAN LEGIRLANGAN Bi_2Te_3 ASOSIDAGI TERMOELEKTRIK MATERIALLARNING ELEKTROFIZIK PARAMETRLARI QIYOSIY TAHLILI
Yusupova D.A.
Sirojiddinova S.Z. **57**

13 EXISTENCE AND UNIQUENESS OF A SOLUTION TO A BITSADZE-SAMARSKII PROBLEM FOR A SUB-DIFFUSION EQUATION WITH THE REGULARIZED PRABHAKAR FRACTIONAL DERIVATIVE
Mirzaeva Maftuna **65**

14 CONVERGENCE OF A NEW TRIVARIATE MITTAG-LEFFLER-TYPE FUNCTION
Tursunova Ergashoy **68**

15 PHYSICS-INFORMED SPECTRAL MEMORY IDENTIFICATION AND FLUTTER-REGIME DIAGNOSTICS FOR A PITCH–PLUNGE AEROELASTIC SYSTEM
Quvvatali Rahimov
Mirzohidjon Rahmatjonov **73**

16 KADMIY TELLURIDGA OLTINGUGURT QO‘SHILGAN HOLDA p-n O‘TISH HOSIL QILISH VA UNING OPTIK XUSUSIYATLARI
Najmidinova Dildoraxon Xalilovna **82**

17 FIZIKA TA'LIMIDA DINAMIK TEST TIZIMINING DIDAKTIK IMKONIYATLARI VA AMALIY AHAMIYATI
Qoraboyev Xusniddin **87**

18 Ni ASOSLI NANOKATALIZATORLARDA UGLEROD NANONAYCHALARINI O‘STIRISH **94**